Time Series Analysis from Observations at Unequal Intervals

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This work is based on a demonstration project published in the *Mathematica* Journal, Volume 7 Issue 1 (Winter 1997). The original was devised for time series Pulsar observations with a resulting twin planet orbit.

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Keith Briggs (University of Western Australia, 1997) suggested the following approach which avoids interpolation and resampling. For each frequency ω that interests you, fit α and β in α sin(ω t)+β cos(ω t) to the data by linear least squares.

The spectral energy $\mathcal{E}[\omega]$ is then the square amplitude integrated over one period, T=2 π/ω:

$$\int_0^{\frac{2\pi}{\omega}} (\alpha \,\mathrm{Sin}[\omega t] + \beta \,\mathrm{Cos}[\omega t])^2 \, dt \Big/ \int_0^{\frac{2\pi}{\omega}} dt \; \text{//Simplify}$$

1/2 (α2+β2)

```
SetDirectory["C:\Documents and Settings\dewr\Desktop"];
```

Load the Lightcurve data

```
raw=ReadList["Zita.txt",Number,RecordLists→True];
```

Find Average values (to be applied to dmag only)

```
c=Apply[Plus,raw]/Length[raw];
```

Reduce time to days starting at 0

```
dT=Table[raw[[n,1]]-raw[[1,1]],{n,1,Length[raw]}];
```

Average magnitude values

```
dmag = Table[raw[[n,2]]-c[[2]],{n,1,Length[raw]}];
```

Recombine the data into a single variable

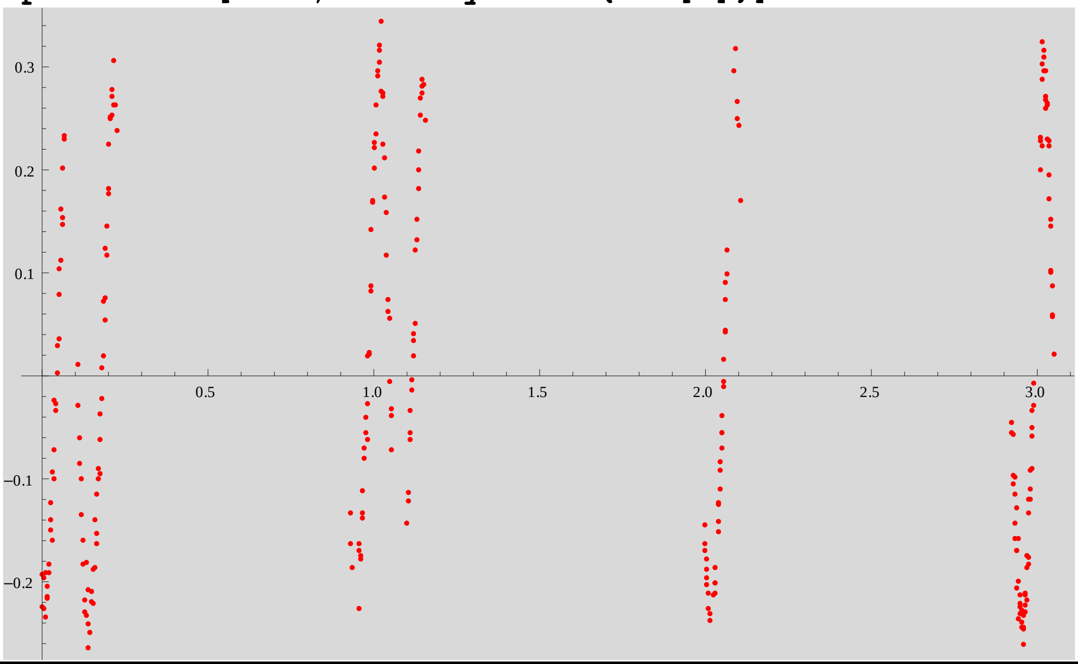
```
data=Table[{dT[[x]],dmag[[x]]}, {x, Length[raw]}];
```

Detertmine Mininmum and Maximum times

```
{tmin,tmax}=First/@{First[data],Last[data]};
```

Now to plot the raw data

```
lp=ListPlot[data, PlotStyle -> {Hue[1]}]
```

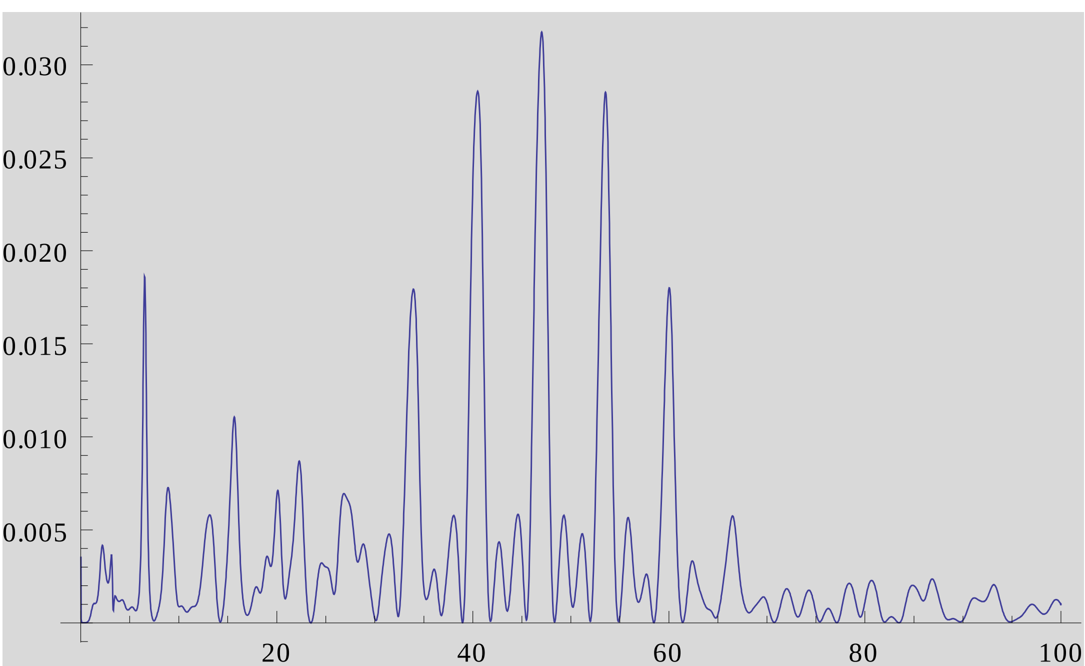


Fit Data to the DFT (I'm assuming it is a Discreet Fourier Transform)

```
ℰ[ω_]:=Fit[data,{Sin[ω t],Cos[ω t]},t]/.
        α_ Sin[_]+β_ Cos[_]:→(α2+β2)/2
```

Now plot the resulting Power Spectrum

```
Plot[ℰ[ω],{ω,0.01,100},PlotRange→All]
```



Find the mimimum value in a selected frequency range (Note this bit doesn't wrk and generates an error in Mathematica Version 6 - awaiting a response from Tech Support)

```
w= ω/.Last[FindMinimum[-ℰ[ω],{ω,23,25}]];
```

As a result we will manually choose a close value from the plot above.

```
w=47
```

We now define a non linear regression

```
basis[α_,β_,θ_,γ_,δ_,ϵ_]=α Sin[θ t]+β Cos[θ t]+ϵ;
```

and load the Mathematica non linear regression package

```
<<"NonlinearRegression`"
```

Pass the data set through the linear regression

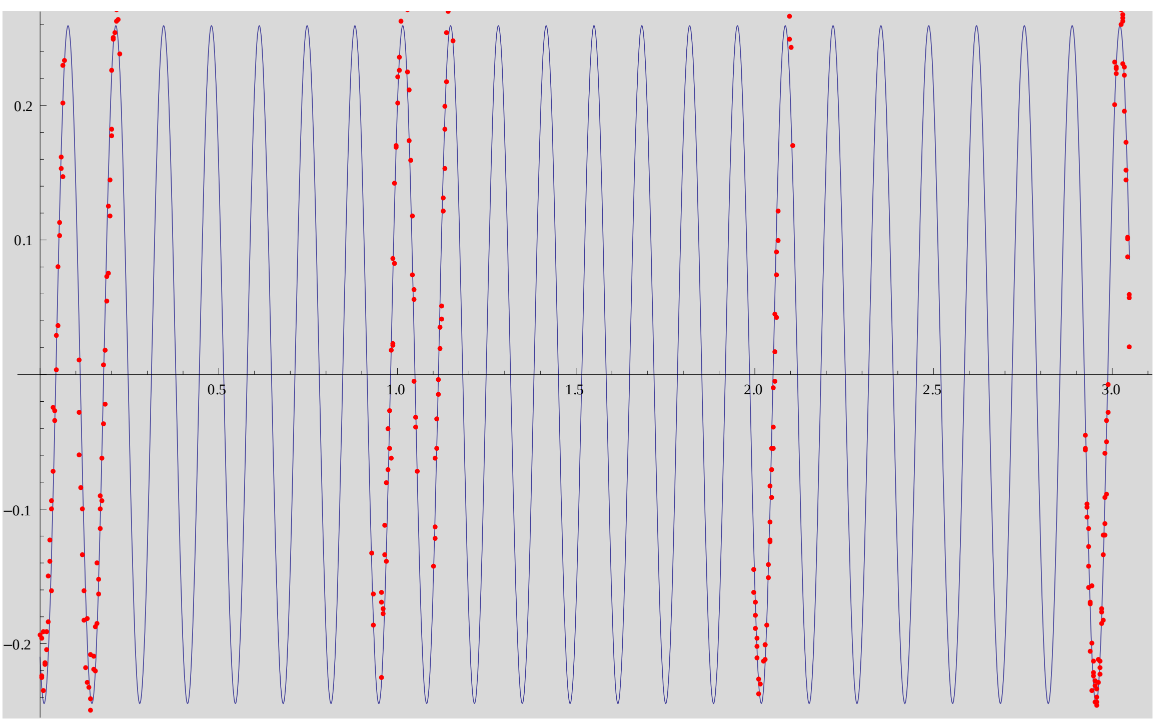
```
md= basis[α,β,θ,γ,δ,ϵ]/.
FindFit[data,basis[α,β,θ,γ,δ,ϵ],{{α,1},{β,1},{γ,1},{δ,0},{θ,w},{
ϵ,0}},t]
```

Output the fitted regression formula, noting that $\theta = 46.9604$ - the actual maximum frequency.

```
0.00743654 -0.217429 Cos[46.9604 t]-0.127139 Sin[46.9604 t]
```

Now lets plot the regression fit to the data

```
Plot[md,{t,tmin,tmax},Epilog ->lp[1]]
```



Wow - it works. Now to convert frequency to Period

```
f=46.9604/(2*Pi); (* Frequency *)
P=2/f ;(* Bimodal Period is 2*1/f *)
```

Now covert the data to produce a phased plot based on the calculated period.

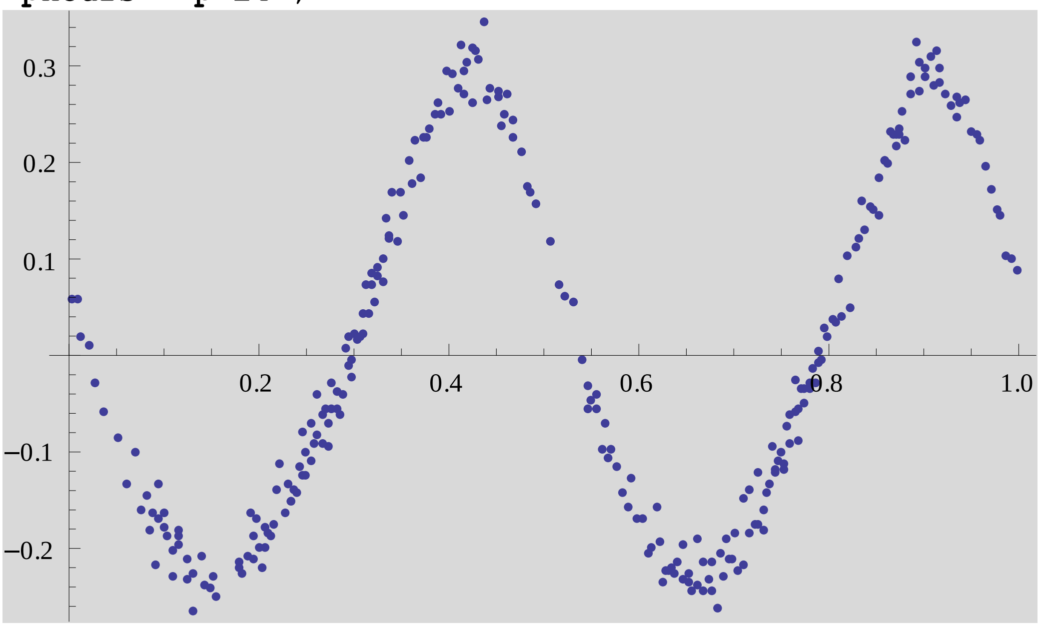
```
phase=Table[(raw[[n,1]]-(IntegerPart[raw[[n,1]]/P]*P))/P,
     {n,1,Length[raw]}];
```

Lets finish up by plotting the phased lightcurve

```
ListPlot[Table[{phase[[i]],dmag[[i]]},{i, 1, Length[raw]}]]
```

And calculate the period

```
pHours = p*24 ;
```



Period

0.267595 Days
6.42228 Hours

Epilogue:

Period calculated in MPO Canopus was slightly different at 6.4248hrs. What needs to be done yet is to devise a method of refining the power spectrum to allow for period analysis over a smaller span of frequencies.

Caveat:

The data loaded into this application may or may not have been light time corrected (maybe accounting for the small period difference between this method in Mathematica and the result from Canopus) BUT it has been pre corrected for variations in magnitude between sessions.